EDUCATION
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Geometry

Altitudes and orthocenter

What is the altitude of a triangle, and where is the orthocenter in acute, right and obtuse triangles? We see it with an animation.

Then the trick for exercises: an altitude makes two right triangles, and in each one the acute angles add up to 90°. We solve 3 exercises on the angles formed by an altitude, step by step.

The method

$$CH \perp AB$$

$$A\hat{C}H = 90^\circ - \hat{A}$$

$$B\hat{C}H = 90^\circ - \hat{B}$$

Worked examples

Exercise 1

In triangle ABC the altitude CH is drawn. Angle A is 30° and angle B is 64°. How big are the angles ACH and BCH?

Data: $CH \perp AB \cdot \hat{A} = 30^\circ \cdot \hat{B} = 64^\circ$

1. **The altitude CH makes two right triangles at H**

$$A\hat{H}C = B\hat{H}C = 90^\circ$$

2. **Triangle AHC: the acute angles add up to 90°**

$$A\hat{C}H = 90° - 30° = 60°$$

Result: $A\hat{C}H = 60°$

3. **Triangle BHC: the same way**

$$B\hat{C}H = 90° - 64° = 26°$$

Result: $B\hat{C}H = 26°$

✓ **Check**

$$\hat{C} = 60° + 26° = 86°$$

$$30° + 64° + 86° = 180° \checkmark$$

Exercise 2

In a right triangle one acute angle is 43°. We draw the altitude to the hypotenuse. Which angles does it make with the legs?

Data: $\hat{C} = 90° \cdot \hat{A} = 43° \cdot CH \perp AB$

1. **The third angle: the acute angles add up to 90°**

$$\hat{B} = 90° - 43° = 47°$$

Result: $\hat{B} = 47°$

2. **Triangle AHC, right-angled at H**

$$A\hat{C}H = 90° - 43° = 47°$$

Result: $A\hat{C}H = 47°$

3. **Triangle BHC, right-angled at H**

$$B\hat{C}H = 90° - 47° = 43°$$

💡 Fun fact: the angles swap! ACH = B and BCH = A

Result: $B\hat{C}H = 43°$

✓ Check

$$47° + 43° = 90° = \hat{C} \checkmark$$

Exercise 3

In triangle ABC: α + β = 112° and α − β = 16°. The altitude AH splits angle α into two parts. How big are they?

Data: $\alpha + \beta = 112° \cdot \alpha - \beta = 16° \cdot AH \perp BC$

1. **Sum and difference: we find α and β**

$$\alpha = (112° + 16°) : 2 = 64°$$

$$\beta = (112° - 16°) : 2 = 48°$$

Result: $\alpha = 64° \quad \beta = 48°$

2. **Triangle ABH, right-angled at H**

$$B\hat{A}H = 90° - 48° = 42°$$

Result: $B\hat{A}H = 42°$

3. **The other part of α**

$$H\hat{A}C = 64° - 42° = 22°$$

Result: $H\hat{A}C = 22°$

✔ **Check**

$$42^\circ + 22^\circ = 64^\circ = \alpha \checkmark$$

$$\gamma = 180^\circ - 112^\circ = 68^\circ$$

$$90^\circ - 68^\circ = 22^\circ \checkmark$$

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